Revenue Maximization of Electric Vehicle Charging Services with Hierarchical Game

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Abstract. Electric Vehicle (EV) industry has ushered in rapid development recently, and many corporations such as Xingxing, State Grid, have built substantial charging stations (CSs) to supply charging services and gain charging revenues. In this paper, we focus on maximizing the sum of the revenues of all CSs managed by one corporation, assuming other corporations' pricing strategies are fixed. We model the EV charging and CS pricing problems as a hierarchical stackelberg game with the corporation at the upper layer as the leader and EV flows at the lower layer as followers. We first analyze the charging strategy equilibrium for EV flows, which however lacks closed-form expressions and thus the backward induction cannot be applied to solve the pricing optimization for the corporation. Therefore, we analyze the hierarchical game as a mathematical program with equilibrium constraints (MPEC). Additionally, a smooth algorithm is applied to solve the MPEC. Simulation results show that the smooth algorithm can achieve high revenues for the corporation.

Keywords: EV flows · Hierarchical game · Pricing optimization · Equilibrium

1 Introduction

In recent years, with the development of fast charging technology, the electric vehicle (EV) industry has ushered in rapid development [1]. As environmentally friendly transportation, EVs have drawn increasing attention from the public and markets [2,3]. Therefore, some corporations such as Xingxing, State Grid, have built substantial charging stations (CSs) to supply charging service in the EV charging market, where corporations compete with each other to gain charging revenues.

As the charging service supplier, corporations are looking for ways to make their business more profitable. There exists competition among different corporations in the charging market, and each corporation wishes to motivate more EVs to charge at CSs managed by it so as to boost revenues. Compared with changing the location of CSs [4], pricing is easier to implement without additional costs. The corporation thus usually adjusts the charging prices for CSs

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overtime to attract more EVs, and a good pricing strategy can make the corporation more profitable. Additionally, as rational individuals, EVs prefer to choose the most suitable CS to minimize charging costs. They take into consideration the charging prices of CSs, distance to CSs, and queuing costs at different CSs while making their charging decisions. Furthermore, as the queuing cost depends on the number of EVs at the same CS, EVs can affect each other's decisions, and there exists a game among different EVs.

Some studies about EV charging and CS pricing have been conducted to maximize the revenues of the CSs [5–9]. However, these works only focus on the revenue of a single CS rather than the revenue of the corporation, and the amount of electric vehicles in these works is small, which is not consistent with the practical applications. In this paper, we consider the pricing optimization for all CSs managed by one corporation to maximize the total revenue of the corporation, assuming other corporations' pricing strategies are fixed. To optimize the pricing strategies for CSs, the corporation should consider the competitors' decisions and anticipate the EVs' charging behavior which may lack closed-form expressions due to the game among EVs. This makes it more challenging to analyze pricing strategies. Additionally, taking into account the great number of EVs in real urban environments further increases the challenge of solving the problem.

To tackle above challenges, we model the EV charging and CS pricing problems as a hierarchical stackelberg game [10]. Specifically, the corporation is the leader in the game, whose goal is to maximize its CSs' total revenues by setting the optimal price for each CS, and its pricing optimization is the upper layer problem. The EVs in the game aim at minimizing the total charging costs. As the pricing optimization is the main issue in the game, we analyze EVs' decisions with a coarser granularity to handle the large number of EVs to solve the hierarchical game effectively. Thus we divide the city into multiple regions, each of which contains a certain number of EVs. We treat the EV flows of regions as followers instead of individual EVs, the EV-flow is a certain number of EVs, whose charging costs optimization is the lower layer problem. As such, the game among EVs becomes the game among EV flows in different regions.

For this hierarchical game, the lower layer problem corresponds to a classical non-cooperative game [11] that is parameterized by the pricing strategies at the upper layer. As the lower equilibrium lacks closed-form expressions, and the upper problem is constrained by it, the upper problem cannot be solved through the classical backward induction method. We instead solve the hierarchical game as a mathematical program with equilibrium constraints (MPEC) [12], through which the hierarchical game can be transformed as a single-level optimization problem. Additionally, we analyze the existence of the equilibrium solution of the hierarchical game, and a smooth algorithm is applied to solve the MPEC. Finally, we compare the smooth algorithm with Block Coordinate Descent (BCD) method [6] and the fixed pricing method which includes the lowest pricing strategy and the highest pricing strategy. The simulation results show that the smooth algorithm can achieve higher revenues for the corporation.

The main contributions of this paper can be summarized as follows. 1) We model the EV charging and CS pricing problems as a hierarchical stackelberg game with the corporation as the leader and the EV flows as followers. 2) We jointly consider the charging costs optimization for EV flows and the pricing optimization for CSs managed by the considered corporation. 3) We analyze the existence of the equilibrium solution of the hierarchical game and formulate the hierarchical game as a MPEC. 4) We apply a smooth algorithm to solve the MPEC, and the simulation results verify the correctness of our theoretical analysis.

2 Related Work

Some previous works have been conducted to explore the EV charging and CS pricing problems. Most existing works mainly focus on two aspects. 1) Maximizing the revenues of the CSs [5–9]. 2) Minimizing the social costs [13–16].

Maximizing the Revenues of the CSs. Jan et al. [5] proposed a dynamic pricing method based on Markov Decision Process (MDP) to maximize the charging service provider's revenues. Cheng et al. [7] proposed a dynamic pricing incentive mechanism to encourage small merchants to install and share their charging equipment with others to adapt to the increasing charging market needs. Wei et al. [8] modeled the CS pricing problem as a multi-leader multi-follower stackelberg game to analyze the price competition among CSs, but the scenario in this paper is only one-dimensional. Woongsup et al. [9] also analyzed the pricing competition among heterogeneous CSs.

Minimizing the Social Costs. Yanhai et al. [13] proposed an algorithm to adjust the prices to incentivize EV flows in different areas to charge at different CSs to minimize the total social costs. Qiang et al. [14] also considered factors such as pricing and distance to model the overall charging problem as an optimization of social welfare. Gagangeet et al. [15] studied the issue of electric energy trading between EVs and CSs in the dynamic pricing charging market, and compared the two cases: modeling EVs as leaders and modeling CSs as leaders. Zeinab et al. [16] proposed a coordinated dynamic pricing model to reduce the overlap between the residential peak power consumption time and the charging station peak power consumption time during the evening peak power consumption period.

However, these works only consider the revenues of a single CS, without considering the corporation's revenues. As one corporation usually manages a certain number of CSs, and the pricing optimization for the multiple CSs is more challenging than for the single CS. Thus the optimization of the corporation's revenues is more intractable but more meaningful. Additionally, these works consider the strategies of individual EVs, but the number of EVs handled is very small, which is not consistent with the actual urban environment.

3 System Model and Game Formulation

3.1 System Model

In this subsection, we introduce the CS pricing and EV charging model. Formally, the city can be abstracted as n regions with N_i, $\forall i \in \{1, 2, ..., n\}$, EVs in region i to be charged, and there are m CSs managed by l corporations. The charging capacity of CS j is N_j^c, $\forall j \in \{1, 2, ..., m\}$, denoting the number of EVs that can be charged by CS j at the same time. Moreover, corporation s, $\forall s \in \{1, 2, ...l\}$, manages H_s CSs, whose charging prices are set by the corporation s, and the revenues of these CSs also belong to this corporation.

In our proposed model, we focus on maximizing the sum of revenues of all CSs managed by one corporation with the charging prices of other corporations' CSs fixed. The corporation can adjust the charging price for each CS dynamically to maximize its revenues by predicting the charging strategies of EV flows in different regions. Subsequently, the EV flows in region i ($\forall i \in \{1, 2, ..., n\}$) can determine the optimal number of EVs to CS j ($\forall j \in \{1, 2, ..., m\}$) based on the price p_j of CS j, the distance d_{ij} to CS j, and the queuing cost q_j at CS j. The goal of EV flows in each region is to ensure that all the charging demands are served and the charging costs are minimized. We then introduce the definition of EV flows' charging costs and the corporation's utility.

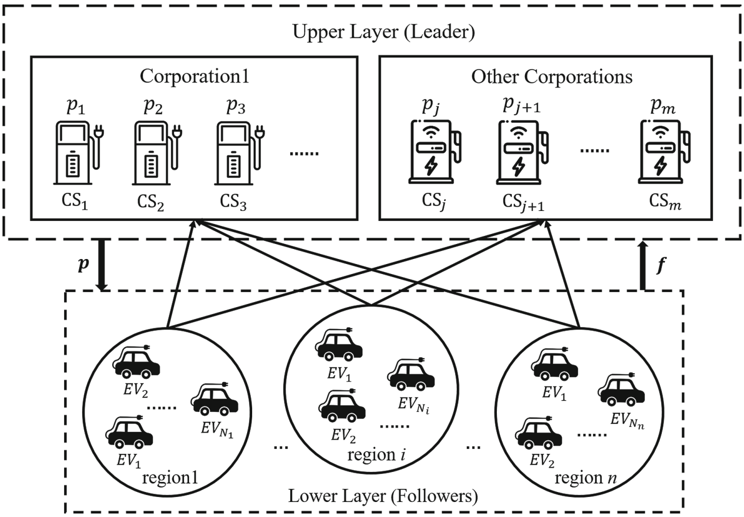

Fig. 1. System model.

EV Flows' Charging Costs. As the EV flows in each region wish to minimize the total charging costs, we first define the cost function of EV flows. Specifically, EV flows make decisions on the number of EVs to each CS according to the estimated charging costs, and the charging costs of EV-flow from region i to CS j is defined as follow

$$C_{ij} = (\omega_1 p_j + \omega_2 q_j + \omega_3 d_{ij}) f_{ij}, \tag{1}$$

where p_j denotes the charging price of CS j, q_j denotes the queuing cost of EV-flow at CS j, and d_{ij} denotes the distance from region i to CS j. Additionally, ω_1, ω_2, and ω_3 are weights assigned to the three types of costs respectively, and f_{ij} denotes the EV-flow from region i to CS j, $0 \leq f_{ij} \leq N_i$. Specifically, the queuing cost depends on the capacity of the CS and the number of EVs that come to the CS, we give a linear assumption about the queuing cost based on [13], which is defined as

$$q_j = \frac{f_j}{N_j^c} = \frac{\sum_{i=1}^{n} f_{ij}}{N_j^c}, \tag{2}$$

where f_j denotes the total EV flows coming to CS j from all the regions, which can be further denoted by $\sum_{i=1}^{n} f_{ij}$. Without loss of generalization, all the EV flows in each region choose different CSs rationally, and we define the cost function of EV flows in region i as

$$C_i = \sum_{j=1}^{m} C_{ij}. \tag{3}$$

Corporation's Utility. The goal of the corporation is to maximize its charging revenue, which comes from the CSs managed by it. For a single CS j, the utility is defined as

$$V_j = (p_j - \varepsilon_j) f_j, \tag{4}$$

where ε_j denotes the average operating cost at CS j. As CSs are managed by different corporations, we assume that the corporation s manages H_s charging stations, H_s should be less than m, and the corporation expects to maximize its revenues. Formally, the utility of corporation s is defined as follow

$$V_s = \sum_{j=1}^{H_s} V_j = \sum_{j=1}^{H_s} (p_j - \varepsilon_j) f_j. \tag{5}$$

3.2 Game Formulation

In this subsection, we investigate the interaction between the corporation and EV flows from a distributed perspective. Specifically, we adopt the stackelberg game which features the hierarchical game structure where the corporation is the leader at the upper layer and the EV flows are followers at the lower layer. We jointly consider the charging costs optimization for EV flows, and the pricing optimization for CSs managed by the corporation. Additionally, we also present the analysis regarding the structure of the game equilibrium.

In a distributed perspective, the EV flows and the corporation in the model determine their optimal strategies based on their interest. The corporation can determine pricing strategies for CSs managed by it, and the EV flows can determine their charging strategies by choosing different CSs. In the proposed game, the corporation is the leader and takes action first in the competition. The corporation can adjust CSs' charging prices first by anticipating the responses of

EV flows' charging behaviors. Upon receipt of all CSs' pricing strategy $\boldsymbol{p}$, EV flows in each region can react to the leader's action and determine their charging strategies. Following the discussions above, we can specify the optimization problem for each participant in the game.

Corporation's Optimization Problem. At the upper layer, the corporation should anticipate the response strategies of the EV flows and consider the pricing strategies of the CSs managed by other corporations to set prices. According to the utility function (5), the optimization problem for corporation s, $\forall s \in \{1, 2, ..., l\}$, can be correspondingly given as

$$\begin{aligned} \max \quad & V_s = \sum_{j=1}^{H_s} (p_j - \varepsilon_j) f_j \\ s.t. \quad & \varepsilon_j < p_j \le p_j^{max}, \end{aligned} \tag{6}$$

where the charging price p_j of CS j, $\forall j \in \{1, 2, ..., H_s\}$, should be larger than the operating cost ε_j due to the individual rationality of the corporation. Additionally, each CS's charging price cannot increase indefinitely due to government management, which means that there is a price ceiling p_j^{max}.

EV Flows' Optimization Problem. At the lower layer, EV flows in region i, $\forall i \in \{1, 2, ..., n\}$, aim at minimizing their total charging costs. As the pricing mechanism provides a simple but effective way to control the EV flows' behavior, EV flows in region i, $\forall i \in \{1, 2, ..., n\}$, can decide on the optimal EV-flow f_{ij} to each CS j based on the CSs' pricing strategy $\boldsymbol{p}$. According to the cost function (3), the optimization problem for EV flows in region i, can be given as

$$\begin{aligned} \min \quad & C_i = \sum_{j=1}^{m} C_{ij} \\ s.t. \quad & \begin{cases} \sum_{j=1}^{m} f_{ij} = N_i, \\ f_{ij} \ge 0, \forall j \in \{1, 2, ..., m\}, \end{cases} \end{aligned} \tag{7}$$

where the constraint in (7) means that all the charging demand in each region should be served.

Lower Equilibrium Condition. As the goal of EV flows is to minimize total charging costs, their decisions are affected by each other due to the queuing cost in the CS. Therefore, the solution to the lower problem (7) can be characterized by the equilibrium. In an equilibrium state, no EV flows can decrease their charging costs by unilaterally changing their charging strategy. Specifically, this concept can be denoted as

$$C(\boldsymbol{f_i^*}) \le C(\boldsymbol{f_i}), \qquad \forall i \in \{1, 2, ..., n\}, \tag{8}$$

where $\boldsymbol{f_i^*}$ denotes the optimal charging strategy of EV flows in region i, i.e., $\boldsymbol{f_i^*} = [f_{i1}^*, f_{i2}^*, ..., f_{im}^*]^T$.

Reformulation of Corporation's Optimization. We have formulated the two-layer hierarchical game, which is constituted by the problem (6) at the leader and (7) at each follower. We have also specified the lower equilibrium condition. We then consider the upper problem, which handles one single optimization problem as we assume that there is only one corporation adjusting its CSs' pricing strategies while other corporations' strategies are fixed. Since the corporation needs to consider the response of the EV flows in different regions, the corporation's optimization can be reformulated as follow

$$\max \quad V_s = \sum_{j=1}^{H_s} (p_j - \varepsilon_j) f_j$$
$$s.t. \quad \begin{cases} \varepsilon_j < p_j \leq p_j^{max}, \\ \boldsymbol{f}_i = \arg\min C_i(\boldsymbol{f}_i), \quad \forall i \in \{1, 2, ..., n\}, \\ s.t. \begin{cases} \sum_{j=1}^{m} f_{ij} = N_i, \\ f_{ij} \geq 0, \forall j \in \{1, 2, ..., m\}, \end{cases} \end{cases} \tag{9}$$

where the lower layer equilibrium condition in (8) is a constraint of the upper optimization problem in (6).

4 Game Analysis and Solving as MPEC

4.1 Game Analysis

In this subsection, we first analyze the lower layer game for EV flows and the equilibrium of the hierarchical game. Then we analyze the optimality conditions at each follower and apply the Karush-Kuhn-Tucker (KKT) condition [17] to present them. Subsequently, we reformulate the hierarchical game as a MPEC. For the lower game, we have the following proposition.

Proposition 1 (Equilibrium of Lower Game). *There always exists a unique equilibrium in the lower game, and the optimization problem of each follower can converge to a unique solution in the equilibrium state, regardless of the pricing strategy* $\boldsymbol{p}$ *at the upper layer.*

Proof. The objective function of problem (7) is continuous, and the inequality and equality constraints are convex. Therefore, the feasible sets of (7) are closed, nonempty, and convex. The Hessian matrix of the utility function C_i is positive definite, which means that $\nabla^2 C_i \succ 0$. Therefore, the utility function C_i is strictly convex, therefore, the lower layer game always exists a unique equilibrium, and the optimization problem of each follower can converge to a unique solution in the equilibrium state, regardless of the pricing strategy $\boldsymbol{p}$ at the upper layer [11]. □

We have proved that the lower game always admits a unique equilibrium, which ensures that the optimization problem of each follower can converge to a unique solution in the equilibrium state when the leader's pricing strategy is given. For the equilibrium of the hierarchical game and the solution of the upper layer problem, we have the following proposition.

Proposition 2 (Equilibrium of Hierarchical Game). *There always exists an equilibrium in the hierarchical game, and the hierarchical game can always converge to a solution which can be denoted as* $[\boldsymbol{p}^*, \boldsymbol{f}^*]$*, where* $\boldsymbol{p}^*$ *is the solution of the upper problem, and* $\boldsymbol{f}^*$ *is the equilibrium solution of the lower game among EV flows.*

Proof. The feasible sets of the problem (9) are continuous, bounded, and convex. The objective function of the corporation is subject to $\boldsymbol{p}$ and the lower equilibrium $\boldsymbol{f}^*$, which is well-defined. For each CS, when the charging price is high, the EVs will become conservative in choosing that CS. Instead, when the charging price is low, the revenues may also be low due to the low charging price. Thus the optimal charging price $\boldsymbol{p}^*$ always exists. Additionally, since the lower game always admits a unique equilibrium, we can infer that there always exists an equilibrium in the hierarchical game, and the hierarchical game can always converge to a solution. □

We have analyzed the characteristics of the hierarchical game. Due to the complicated game among the followers and the lack of closed-form equilibrium expressions in the lower game, the corporation's optimization problem cannot be solved through the classical backward induction method. Thus we solve the hierarchical game as a MPEC. As the best-response at each follower corresponds to a concave problem, we can apply the KKT condition [17] to equivalently present the optimality conditions $\boldsymbol{f}_i = \arg\min C_i(\boldsymbol{f}_i)$ at each follower. Therefore, the problem (9) can be reformulated as

$$\max \quad V_s = \sum_{j=1}^{H_s}(p_j - \varepsilon_j)f_j$$
$$s.t. \quad \begin{cases} \varepsilon_j < p_j \leq p_j^{max}, \\ \nabla_{\boldsymbol{f}_i} L_i = 0, \quad \forall i \in \{1, 2, ..., n\}, \\ \sum_{j=1}^{m} f_{ij} = N_i, \quad \forall i \in \{1, 2, ..., n\}, \\ \nu_{ij} f_{ij} = 0, \quad \forall i \in \{1, 2, ..., n\}, \forall j \in \{1, 2, ..., m\}, \\ \nu_{ij} \geq 0, \quad \forall i \in \{1, 2, ..., n\}, \forall j \in \{1, 2, ..., m\}, \end{cases} \tag{10}$$

where L_i is the Lagrange function at the follower-i, given as

$$L_i(\boldsymbol{f}_i, \boldsymbol{\nu}_i, \lambda_i) = C_i - \boldsymbol{\nu}_i \boldsymbol{f}_i + \lambda_i(N_i - \sum_{j=1}^{m} f_{ij}), \tag{11}$$

where $\boldsymbol{\nu}_i = [\nu_{i1}, \nu_{i2}, ..., \nu_{im}]^T$ and λ_i are the corresponding Lagrange multipliers. As the constraints $\nu_{ij} f_{ij} = 0$ are complementary constraints which are very complicated and hard to handle, it is intractable to solve the MPEC directly. Additionally, if we enumerate each combination situation corresponding to the complementary constraints, although the problem in each situation is convex, there will be 2^{m*n} problems to be solved, which is extremely inefficient. Therefore, we perturb the original MPEC following [18] and obtain a sequence of smooth problems, the solutions of which converge to a solution of the original MPEC. The details of the smooth algorithm are presented in the next subsection.

4.2 Solution for MPEC

In this subsection, we perturb the original MPEC and consider a sequence of smooth and regular problems as stated above. Specifically, we consider the perturbed problem $\boldsymbol{P}(\mu)$ with parameter μ as follow:

$$\max \quad V_s = \sum_{j=1}^{H_s} (p_j - \varepsilon_j) f_j$$
$$s.t. \quad \begin{cases} \varepsilon_j < p_j \le p_j^{max}, \\ \nabla_{\boldsymbol{f}_i} L_i = 0, \quad \forall i \in \{1, 2, ..., n\}, \\ \sum_{j=1}^{m} f_{ij} = N_i \quad \forall i \in \{1, 2, ..., n\}, \\ \nu_{ij} \ge 0, \quad \forall i \in \{1, 2, ..., n\}, \ \forall j \in \{1, 2, ..., m\}, \\ f_{ij} - z_{ij} = 0, \quad \forall i \in \{1, 2, ..., n\}, \ \forall j \in \{1, 2, ..., m\}, \\ \sqrt{(z_{ij} - \nu_{ij})^2 + 4\mu^2} - (z_{ij} + \nu_{ij}) = 0, \end{cases} \tag{12}$$

where $\boldsymbol{z} \in \Re^{m*n}$ is an auxiliary variable. We can observe that the complementary constraints in (10) are replaced by the last two constraints in $\boldsymbol{P}(\mu)$. Specifically, when $\mu = 0$, the last constraint in $\boldsymbol{P}(\mu)$ may reduce to two cases: (1) $\nu_{ij} = 0$ and $\sqrt{z_{ij}^2} - z_{ij} = 0$, (2) $f_{ij} = 0$ and $\sqrt{(-\nu_{ij})^2} - \nu_{ij} = 0$. Since these two cases correspond to the complementary constraints in (10), $\boldsymbol{P}(\mu)$ is equal to the original MPEC. When $\mu > 0$, $\boldsymbol{P}(\mu)$ is a well-defined smooth problem, and it can be solved by standard optimization tools. When $\mu \to 0$, the solution of the MPEC will converge to a stationary point by [18]. The smooth algorithm is shown in Algorithm 1.

Algorithm 1. Smooth Algorithm for MPEC

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1: Let {μ^k} be any sequence of nonzero numbers with lim_{k→∞} μ^k = 0;
2: choose ω^0 = (p^0, f^0, ν^0, z^0, λ^0) ∈ ℜ^{H_s+3m*n+n}, and set k = 1;
3: while ||e|| > ϵ do
4:     Find a stationary point ω^k of P(μ^k);
5:     e = ω^k − ω^{k−1};
6:     Set k = k + 1
7: end while;
```

As shown in Algorithm 1, we can solve a sequence of problems $\boldsymbol{P}(\mu)$ to obtain a solution of the original MPEC. As the pricing of CSs, the decisions of EV flows, the Lagrange multipliers, and the auxiliary variable are all variables in $\boldsymbol{P}(\mu)$, we choose an initial value of these variables $\omega^0 = (\boldsymbol{p}^0, \boldsymbol{f}^0, \boldsymbol{\nu}^0, \boldsymbol{z}^0, \boldsymbol{\lambda}^0)$ and solve $\boldsymbol{P}(\mu)$ by standard optimization tools. Then we calculate the Euclidean distance between two iterations which is denoted by $\|e\|$. When $\|e\|$ is lower than a threshold ϵ, the algorithm stops. Specifically, as μ is also a parameter of $\boldsymbol{P}(\mu)$, and for $\lim_{k\to\infty} \mu^k = 0$, the parameter μ should be initially set to a number which is close to 0 (e.g., 0.0001) and reduced at each iteration.

Table 1. The experimental parameters

Parameters	Description	Value	Parameters	Description	Value
ω_1, ω_2, ω_3	Different weights	0.6, 0.1, 0.3	μ	Algorithm parameter	0–10^{-4}
N_j^c	Charging capacity	4–32 piles	ϵ	Stopping parameter	10^{-4}
p_j	Charging price	0.8–1.8 RMB	n	Region number	11, 13
d_{ij}	Distance to CS	1–15 km	ε_j	Operating cost	0.2–0.4 RMB

5 Simulation Results

In this section, we design a simulation to validate the effectiveness of the model and the smooth algorithm. All computations are performed on a 64-bit machine with 16 GB RAM and a six-core Intel i7-8770 3.20 GHz processor. And the optimization problems are solved with python 3.8, scipy 1.5.2 and pyomo 5.7.3.

To better imitate the real environment, we collect the EVs and CSs data in Nanjing (NJ) and Wuhan (WH)[1]. We then generated some summary statistics based on real data in the two cities. We divide Nanjing into 11 regions and divide Wuhan into 13 regions according to the official administration information, and all CSs in one region are abstracted as one CS for simplicity. The number of EVs in Nanjing is about 130000, we suppose 5% of the 130000 EVs (6500 EVs) that charge in the charging game as [13] in Nanjing, and 4000 EVs (5% of the 80000 EVs) charge in the charging game in Wuhan. We assume that the EVs' distribution follows the residential population distribution, and the result of the two cities is shown in Fig. 2 and Fig. 3. The distance from each region to each CS is obtained according to the Google traffic map. Additionally, according to the data on Telaidian (www.teld.cn), the charging capacity at different CSs fluctuates from 4 to 32, and the pricing of each kilowatt-hour of electricity at different CSs fluctuates from 0.8 to 1.8 RMB. Thus the average charging fee of each EV when fully charged can be estimated within the range of 40 to 90 RMB, and we assume the operating cost is a quarter of the charging price. Additionally, the weights in (1) are set as $\omega_1 = 0.6$, $\omega_2 = 0.1$, $\omega_3 = 0.3$ as [13]. The parameter μ is initially set to 0.0001 and reduced by a factor of 100 at each iteration, and the stopping parameter ϵ is set to 10^{-4} as [18]. The experimental parameters are listed in Table 1.

We compare the smooth algorithm with Block Coordinate Descent (BCD) method [6] and the fixed pricing method which includes the lowest pricing strategy (PFix-min) and the highest pricing strategy (PFix-max). All the evaluations are performed based on the data generated from the real data in Nanjing and Wuhan as stated above. We first compare the running time of the smooth algorithm with the BCD algorithm based on the data in Nanjing and Wuhan. The results are shown in Fig. 4 and Fig. 5, from which we can observe that the smooth algorithm is faster than BCD algorithm obviously, and as the scale of the problem increases, the distinction becomes more obvious.

[1] http://www.cheyanjiu.com/info.php?CateId=19.

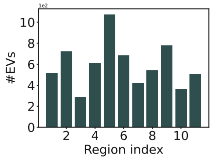

Fig. 2. EVs distribution (NJ)

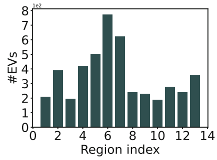

Fig. 3. EVs distribution (WH)

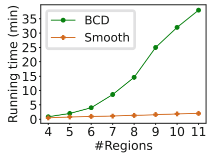

Fig. 4. Running time (NJ)

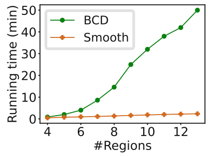

Fig. 5. Running time (WH)

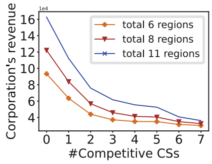

Fig. 6. Effects of competitive CSs (NJ)

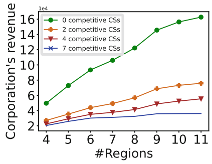

Fig. 7. Effects of regions (NJ)

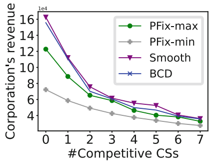

Fig. 8. Algo. Comp. w.r.t. #CSs (NJ)

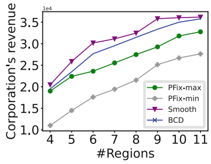

Fig. 9. Algo. Comp. w.r.t. #regions (NJ)

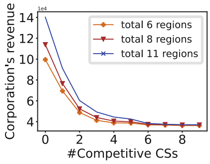

Fig. 10. Effects of competitive CSs (WH)

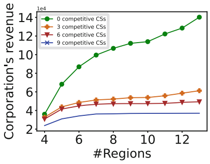

Fig. 11. Effects of regions (WH)

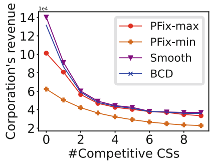

Fig. 12. Algo. Comp. w.r.t. #CSs (WH)

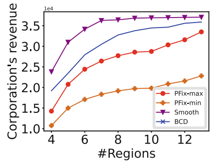

Fig. 13. Algo. Comp. w.r.t. #regions (WH)

We then investigate the corporation's revenue based on the data in Nanjing. We first investigate the corporation's revenue calculated by the smooth algorithm under different number of total regions. As shown in Fig. 6, as the number of competitive CSs increases, the corporation's revenue becomes lower, because more CSs participate in the competition. We then investigate the corporation's revenue under the different numbers of competitive CSs. The result is given in Fig. 7, from which we can observe that when there are more regions considered, the CSs will be more heavily loaded, and the corporation has a higher chance to improve the total revenue. We then compare the smooth algorithm with the algorithms mentioned above. As shown in Fig. 8 and Fig. 9, the smooth algorithm can achieve higher revenue for the corporation in the cases where there are different number of competitive CSs and regions.

We then investigate the corporation's revenue based on the data in Wuhan. We also investigate the corporation's revenue under different cases. As shown in Fig. 10, as the number of competitive CSs increases, the corporation's revenue becomes lower. Figure 11 shows that when there are more regions considered, the corporation's revenue will be higher. The results are the same as the results in Nanjing. We then compare the smooth algorithm with other algorithms, Fig. 12 shows that the smooth algorithm can achieve higher revenue than other algorithms when the number of competitive CSs changes, and Fig. 13 shows that the smooth algorithm can achieve higher revenue for the corporation when the number of regions changes. All results above show the effectiveness of the smooth algorithm in our model.

6 Conclusion and Future Work

In this paper, we propose a hierarchical stackelberg game to investigate the EV charging market where CSs are managed by different corporations. Specifially, we study the pricing optimization problem for one corporation assuming other corporations' pricing strategies are fixed. In the proposed game, the corporation is the leader, whose goal is to maximize its total revenue by setting the most suitable price for each CS managed by it. To handle a large number of EVs in the urban environment, we treat the EV flows as followers instead of individual EVs. The EV flows can decide charging behavior to minimize their total charging costs. Due to the lack of closed-form expressions of the lower equilibrium, we analyze the hierarchical game as a MPEC and apply the smooth algorithm to find the solution for the MPEC. Simulation results have shown that the smooth algorithm can achieve high revenues for the corporation.

As the future work, we will investigate the equilibrium among different corporations, assuming all corporations can adjust the pricing strategies. This can be modeled as an equilibrium problem with equilibrium constraints (EPEC). Diagonalization methods have been widely used by researchers in engineering fields to solve EPECs [19], which inspires us to solve the equilibrium among different corporations.

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